The complete census of orientable cusped hyperbolic 3-manifolds, up to 11 tetrahedra

Shana Y. Li

University of Illinois, Urbana-Champaign

February 2026

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Crossings	Knots		
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5		5_1	5_2
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$$\downarrow$$

minimal number of tetrahedra in triangulations of manifolds

Tetrahedra	Orientable cusped hyperbolic 3-manifolds				
2				m003	m004
3	m006	m007	m009	m010	m011
	m015	m016	m017	m019	

Table: The census of orientable cusped hyperbolic manifolds up to 3 tetrahedra

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Burton, 2014

There are precisely 75,956 cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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L. 2025

There are precisely 150,730 and 505,352 orientable cusped hyperbolic 3-manifolds whose minimal ideal triangulations consist of 10 and 11 tetrahedra respectively.

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3-step outline of creating a complete census:

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3. Group eligible candidates into isomorphism classes and certify the distinctness of each class

For 10 tetrahedra census:

Step	Candidates	Discarded	Kept
Candidate generation	8,373,308	0	0

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Certify hyperbolicity ($r = 60$)	3269	0	30,538
Exhaustive nonminimality ($h = 5$ and 6)	6	3263	0

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Certify hyperbolicity ($r = 1000$)	0	0	6

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Candidate generation	8,373,308	0	0
Greedy nonminimality	1,946,782	6,426,526	0
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Exhaustive nonminimality ($h = 2$)	698,650	374,224	0
Special planes	33,807	664,843	0
Certify hyperbolicity ($r = 60$)	3269	0	30,538
Exhaustive nonminimality ($h = 5$ and 6)	6	3263	0
Certify hyperbolicity ($r = 1000$)	0	0	6
Total	8,373,308	7,468,856	904,452

For 11 tetrahedra census:

Step	Candidates	Discarded	Kept
Candidate generation	27,794,289	0	0

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Candidate generation	27,794,289	0	0
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Candidate generation	27,794,289	0	0
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Certify hyperbolicity ($r = 1$)	3,019,903	0	2,525,631

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Candidate generation	27,794,289	0	0
Greedy nonminimality	5,545,534	22,248,755	0
Certify hyperbolicity ($r = 1$)	3,019,903	0	2,525,631
Special surfaces	380,143	2,639,760	0

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Exhaustive nonminimality ($h = 2$)	112,502	267,641	0

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Special surfaces	380,143	2,639,760	0
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Certify hyperbolicity ($r = 60$)	15,768	0	96,734

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Exhaustive nonminimality ($h = 5$ and 6)	196	15,572	0

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Certify hyperbolicity ($r = 60$)	15,768	0	96,734
Exhaustive nonminimality ($h = 5$ and 6)	196	15,572	0
Certify hyperbolicity ($r = 10000$)	0	0	196

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Certify hyperbolicity ($r = 1$)	3,019,903	0	2,525,631
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Exhaustive nonminimality ($h = 2$)	112,502	267,641	0
Certify hyperbolicity ($r = 60$)	15,768	0	96,734
Exhaustive nonminimality ($h = 5$ and 6)	196	15,572	0
Certify hyperbolicity ($r = 10000$)	0	0	196
Total	27,794,289	25,171,728	2,622,561

Epstein & Penner, 1988

There is a canonical way to give a cusped hyperbolic 3-manifold a polygonal cellular structure, using its geometric structure.

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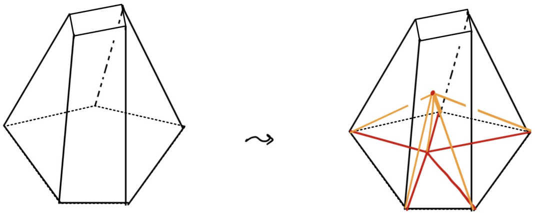

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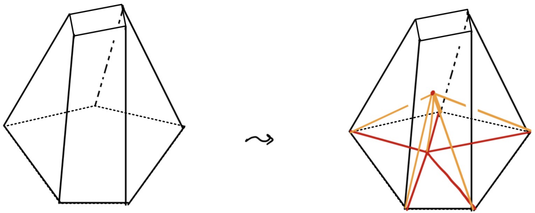

The triangulation obtained is called the *canonical triangulation*.

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Fact

If two cusped hyperbolic 3-manifolds have the same unverified canonical triangulations, they are isometric to each other.

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Fact

If two cusped hyperbolic 3-manifolds have the same unverified canonical triangulations, they are isometric to each other.

This was used to group the candidates into isomorphism classes.

Separate isomorphism classes:

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Drawback: Expensive to compute & Need to go back and forth

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Verified computation always gives correct canonical triangulations, serving as a complete invariant.

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- Within each class: all reprsent the same manifold via unverified canonical triangulations (which do preserve topology).

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L. 2025

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For 11-tetrahedra triangulations: 203/2,622,561 uncomputed. Handled using (volume[:80], H_1, S_2^{ab}).

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There are precisely 505,352 orientable cusped hyperbolic 3-manifolds triangulable by a minimum of 11 tetrahedra.

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Census of exceptional Dehn fillings

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6-Theorem (Agol, 2000)

Let M be a 1-cusped hyperbolic 3-manifold. If a slope (p, q) has length larger than 6, then $M(p, q)$ is hyperbolic.

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A Dehn filling on a cusped hyperbolic 3-manifold is *exceptional* if the resulting manifold is not hyperbolic.

6-Theorem (Agol, 2000)

Let M be a 1-cusped hyperbolic 3-manifold. If a slope (p, q) has length larger than 6, then $M(p, q)$ is hyperbolic.

Corollary

There are only finitely many exceptional Dehn fillings on a 1-cusped hyperbolic 3-manifold.

Dunfield, 2019

There are precisely 205,822 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by no more than 9 tetrahedra.

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Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0

L., 2025

There are precisely 439,898 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimum of 10 tetrahedra.

Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	439,933	360,514	0

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Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	439,933	360,514	0
Regina census lookup	11,540	24	428,369

L., 2025

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Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	439,933	360,514	0
Regina census lookup	11,540	24	428,369
Essential S^2 or torus	11	0	11,529

L., 2025

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Step	Candidates	Discarded	Confirmed
Candidate generation	800,447	0	0
Certify hyperbolicity	439,933	360,514	0
Regina census lookup	11,540	24	428,369
Essential S^2 or torus	11	0	11,529
Identified to be m135(1,3)	0	11	0

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Total	800,447	360,549	439,898

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Essential S^2 or torus	11	0	11,529
Identified to be m135(1,3)	0	11	0
Total	800,447	360,549	439,898

These reveal exactly 1849 knot exteriors in 10-tetrahedra census

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There are precisely 1,340,930 exceptional Dehn fillings on 1-cusped hyperbolic 3-manifolds triangulable by a minimum of 11 tetrahedra.

Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0

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Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0
Regina census lookup	1,249,311	10,799	1,288,713

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Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0
Regina census lookup	1,249,311	10,799	1,288,713
Certify hyperbolicity	52,242	1,197,069	0

L., 2026

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Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0
Regina census lookup	1,249,311	10,799	1,288,713
Certify hyperbolicity	52,242	1,197,069	0
Essential S^2 or torus	25	0	52,217

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Step	Candidates	Discarded	Confirmed
Candidate generation	2,548,823	0	0
Regina census lookup	1,249,311	10,799	1,288,713
Certify hyperbolicity	52,242	1,197,069	0
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These reveal exactly 4673 knot exteriors in 11-tetrahedra census

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- Successfully computed $x(M)$ for all but 21 of them
- Equality holds for all the computed ones

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In particular, they are the smallest among all.

	Tetrahedra									
Cusps	2	3	4	5	6	7	8	9	10	11
1	2	9	52	223	913	3388	12,241	42,279	144,016	482,972
2	0	0	4	11	48	162	591	1934	6585	21,984
3	0	0	0	0	1	2	13	36	123	391
4	0	0	0	0	0	0	1	1	5	5
5	0	0	0	0	0	0	0	0	1	0

The 10-tetrahedra census comes automatically with `SnapPy` 3.3.
The 11-tetrahedra census is available for installation on GitHub. QR code:

Alternatively: shana-y-li.github.io $\rightarrow$ Code $\rightarrow$ snappy_11_tets